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How Indonesian and Singaporean Students (might) Learn the Pythagorean Theorem: A Comparative Analysis of Textbooks

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How Indonesian and Singaporean Students (might) Learn the Pythagorean Theorem: A Comparative Analysis of Textbooks

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Abstract. Textbook is considered as a potentially implemented curriculum because it mediates between the intended curriculum and teachers' classroom practices. Textbooks heavily influence the teaching and learning process that, consequently, implies students' achievement. A number of studies show the relationships between students' mathematics achievement and the textbooks they use during the learning processes. The present study reports a comparative analysis of Indonesian and Singaporean textbooks for the topic of the Pythagorean Theorem. An Indonesian mathematics textbook and a Singaporean mathematics textbook for junior high schools were analyzed. The results of the textbook analysis show that both textbooks used activities-based and context-related explanation section. Although Indonesian and Singaporean textbooks included context, the purpose of context in both textbooks were different.

1. Introduction

Student achievement is often connected to a question whether the students have learned sufficient content and/or competences. This question has led to the emergence of the concept of opportunity-to-learn. In the report of the First International Mathematics Studies [1], opportunity-to-learn was defined as "whether or not […] students have had the opportunity to study a particular topic or learn how to solve a particular type of problem." (pp. 162–163). With respect to the opportunity-to-learn, Liu [2] considered four main areas, i.e. (a) *content coverage* that concerns the match between the curriculum content and the scope of the test, (b) *content exposure* that refers to the time spent on the content that will be tested, (c) *content emphasis* that concerns whether the teachers emphasis only on the content that will be tested, and (d) *quality of instructional delivery* that refers to the sufficiency of teaching the content. A different perspective of opportunity-to-learn was formulized by Brewer and Stasz [3] who distinguished three domains of opportunity-to-learn. The first domain is *the curriculum content* that concerns whether the students have been taught the subjects and topics that are essential to attain the standards. This first domain is quite close to Liu's conception of opportunity-to-learn. The second domain is *the instructional strategies* that refer to teachers' strategies or approaches in delivering the curriculum content. The third category is *the instructional resources* that cover the learning resources that are used by teachers and students, such as textbooks and student worksheets.

Among the three dimensions of opportunity-to-learn proposed by Brewer and Stasz, the instructional resources have a quite special position because this dimension is often seen as the mediator between the first and the second dimensions of opportunity-to-learn. IEA positions textbooks as potentially implemented curriculum because they mediate between the intended curriculum and the instructional

practices of teachers [4]. This view is supported by the findings of a number of studies that the topics taught in the classroom and teachers' teaching strategies are strongly influenced by the textbooks teachers use [5], [6]. Consequently, textbooks are seen as an important factor that determines students' opportunity-to-learn [7], [8]. A lot of studies have shown the relationship between textbooks and students' mathematics performance. Some of these studies connected a textbook analysis with the learning of particular mathematics topics, such as fractions (see [9], [10]) and subtraction [11]. Some other textbook analyses paid attention to the learning of higher order thinking skills, such as problem solving skills (e.g. [12], [13]), and mathematical modeling (e.g. [14], [15]). Textbook analysis also covers the learning of mathematical symbol. For example, the studies of Li, Ding, Capraro, and Capraro [16] and McNeil et al. [17] revealed that students' perception and conception of the equal sign are influenced by how the textbooks positioned the equal sign, which was mainly merely as symbol of 'result of operation' rather than on 'equality'.

Different textbooks might provide different opportunity-to-learn for students [18]. A nice of example of textbook comparison was performed by Haggarty and Pepin [18] who analyzed the characteristics of mathematics textbooks in English, French, and German mathematics classrooms. They found that the majority of questions in English textbooks were straightforward applications of worked examples. These tasks mostly did not use real contexts and required lower levels of thinking. In contrast, German mathematics textbooks mainly provided tasks with high level of complexity in terms of mathematical logic and structure. Such finding supports the idea that analyzing textbooks from different countries can be used to find an explanation for the differences of students' mathematics performance across countries [4]. Taking this idea, Gatabi and Stacey [14] analyzed Iranian and Australian mathematics textbooks to investigate the characteristics of mathematics tasks in these textbooks with respect to promoting mathematical literacy. They found that in comparison to the Australia textbooks, the Iranian textbooks contained fewer problems with a diverse range of contexts and real-world application. In conclusion, the Iranian textbooks provided little opportunity for students to do mathematical modelling, which is seen as the key process of mathematical literacy. Various important findings of textbook comparisons have led the present study to investigate the opportunity-to-learn the Pythagorean Theorem that is provided by Indonesian and Singaporean mathematics textbooks. Singapore is taken into consideration for the comparison because Singapore is well known for its high mathematics performance. Furthermore, a topic on the Pythagorean Theorem becomes a point of interest because this theorem has a variety of proofs that could be used for classroom activities.

In the Trend of International Mathematics and Science Studies (TIMSS), textbooks are analyzed based on five measures [4]. The first measure deals with the classroom activities which are implied by the textbook. The second measure concerns the amount of content covered in textbooks. The third measure focuses on the sequencing of textbook content. The fourth measure deals with the physical characteristics of textbooks, such as the size of the book and the number of pages. Lastly, the fifth measure characterizes the cognitive demands of textbook content. Another approach to analyze textbooks was proposed by Pepin and Haggarty [19] who distinguished four areas of analysis. These areas include (1) the mathematics topics presented in textbooks and the underlying beliefs about the nature of mathematics, (2) the methods to help students understand the textbooks' content, (3) the sociological contexts of textbooks which concerns whether textbooks are adaptive to different levels of students' ability, and (4) the cultural traditions in textbooks. A different distinction for textbook analysis was coined by Charalambous et al. [9] who classified three foci of textbook analysis, i.e. horizontal, vertical, and contextual. The horizontal analysis covers the general characteristics of textbooks, such as physical characteristics and the organization of the content. The result of this analysis gives early impression of opportunity-to-learn because it provides information on the exposure of textbooks' content. Nevertheless, information about the quality and the didactical aspects of the textbooks' content is not revealed by a horizontal analysis. Therefore, a vertical analysis is needed to address how textbooks present and treat the content. Such an analysis offers an in-depth understanding of the mathematical content. The third category, the contextual analysis, focuses on how textbooks are used in instructional

activities. Therefore, Charalambous et al. [9] argued that, in fact, only the first two categories are appropriate to analyze the characteristics of textbooks.

2. Method

The main focus of the present study was on how the Pythagorean Theorem was presentd in Indonesian and Singaporean textbooks. The Pythagorean Theorem was taught in Grade 7, both in Indonesia and Singapore. Therefore, grade 7 mathematics textbooks were selected for the present study, i.e. *New Syllabus in Mathematics* (Singapore) and *Matematika untuk SMP* (Indonesia). Both textbooks were published by private publisher.

Regarding the analysis, the present study used Charalambous et al.'s [9] vertical analysis because the main point of the textbook analysis was identifying the content delivery of the two textbooks. The vertical analysis was meant to investigate the opportunity to learn the Pythagorean Theorem provided by the two textbooks. In particular, the textbook analysis concerns four main aspects, i.e. the structure of content delivery, the use of context, application problem, and suggestion for technology. The structure of content delivery covered such whether the textbook directly starts from a given formula or from activities. The use of context concerns whether the textbooks use context as a starting point to introduce the concept, whereas application problem deals with the use of application problem in the exercises. Lastly, suggestion for technology refers to the integration of information, communication, and technology (ICT) into the activities in the textbooks.

The first phase of the analysis was looking at the book in general view by focusing on the main features of the textbooks. In the next phase, the analysis was done in a narrower scope for which units of analysis were set. There were three types of unit analysis, i.e. content section, worked examples, exercises, and supplementary materials.

3. Results and Discussion

The first focus of the textbook analysis was the features of the textbooks. As indicated on its front pages, the Singaporean textbook contained five main features, i.e. investigation, class discussion, thinking time, journal writing, and performance task (see Figure 1). 'Investigation' referred to the activities to be done by students in order to discover and learn the mathematics concepts. 'Class discussion' was the textbook section that contained questions to discuss by students. Students were also provided opportunity to further develop their thinking through 'Thinking time' section. After students grasped the mathematics concepts, they were encouraged to reflect on their thinking in the 'Journal writing' section. Lastly, exercises were provided in the 'Performance task' section for which students could practice what they have learned. Unlike the Singaporean textbook, the Indonesian textbook did not provide detailed information about its main features. Nevertheless, the Indonesian textbook gave information about the types of tasks that were included in the book (Figure 2). The first task type was indicated with a bulb icon and was aimed to focus on students' conceptual understanding. The second task type dealt with application problems and/or tasks that required students' mathematical communication. This task type was marked with mouse icon. The last task type was problem solving tasks or higher order thinking tasks that was indicated with the head or brain icon.
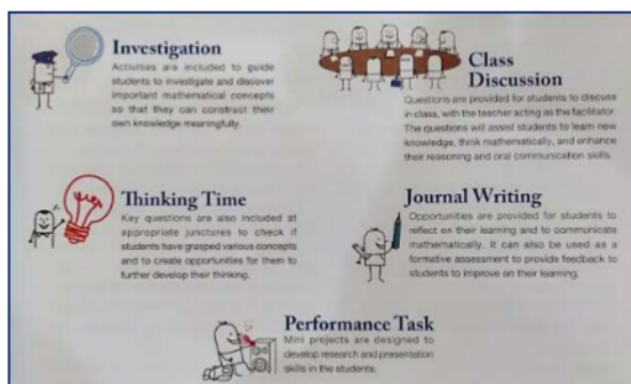
Figure 1. The main features of the Singaporean textbook (*source: a scanned page of New Syllabus in Mathematics*)
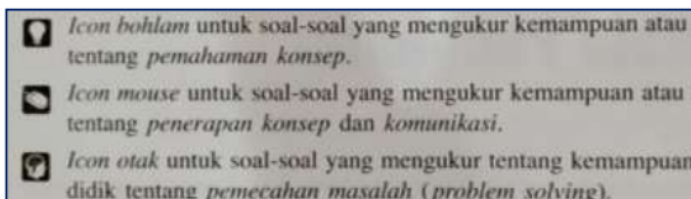

Figure 2. The types of tasks in the Indonesian textbook (*source: a scanned page of Matematika untuk SMP*)

After comparing the main features of the textbooks, the textbook analysis focused on the main structure of the content delivery. It was found that both textbooks used 'activity-based explanation'. In such approach the Pythagorean Theorem was not directly given to students, but it was obtained by students through discovery or investigation learning activities. The two textbooks used different approach for students' activities in discovering the Pythagorean Theorem. The Singaporean textbook used length measurement approach, whereas the Indonesian textbooks used area measurement approach. In the Singaporean textbook, students were provided with a set of right-angled triangles ABC with a given side (see the left figure on Figure 3). Students were asked to measure the length of the other sides of the triangles. After students got the measures of all sides, they wrote the length of the sides in a given table. The table also contained the columns AB^2, AC^2, and BC^2 for students to fill in. The Pythagorean Theorem was obtained by investigating the relationship between the squares of the three sides of all right-angled triangles. A hint was provided for students to do the investigation, i.e. a column $BC^2 + AC^2$. This column was aimed to guide students that the required relationship involved the sum of the squares of two sides of a right-angled triangle. Measurement activities was also used in the Indonesian textbook to discover the Pythagorean Theorem. However, the Indonesian textbook used area measurement; in particular the area of squares (see the right figure on Figure 3). The Indonesian textbook provided students with a set of right-angled triangles with a square was attached to each side of the triangles. The side of a square had the same length with the corresponding side of a triangle. Students were asked to determine the area of all squares on the perimeter of a right-angle triangle. Similar to the Singaporean textbook, the Indonesian textbook also provided a table to write the measured areas. The Pythagorean Theorem was obtained by investigating the relationship between the areas of squares on the three sides of all right-angled triangles. The emergence of AB^2, AC^2, and BC^2 in the Indonesian textbook was quite natural because they were the areas of squares. In contrast, in the Singaporean textbook AB^2, AC^2, and BC^2 suddenly appeared without any explanation so that the students might not fully understand the meanings of AB^2, AC^2, and BC^2.
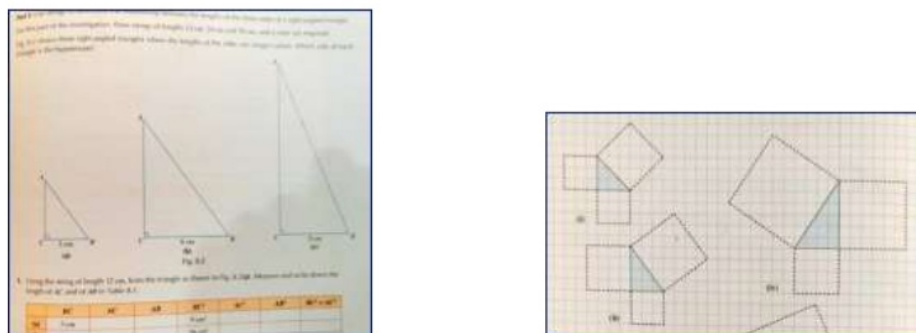
Figure 3. A length measurement approach in the Singaporean textbook (left) and an area measurement approach in the Indonesian textbook (right)
(*source: a scanned page of New Syllabus in Mathematics and Matematika untuk SMP*)

The transition from measuring the length of a triangle's sides – i.e. AB, AC, and BS – to the appearance of AB^2, AC^2, and BC^2 in the Singaporean textbook is not smooth. However, the length measurement activities in the textbook was a smooth transition from the context given at the beginning of the Pythagorean Theorem chapter. As the starting point to introduce the Pythagorean Theorem, the Singaporean textbook used the context of history of mathematics. The textbook informed the reader how the ancient Egyptians used knotted ropes to form right-angled triangles (see the left figure on Figure 4). These people knew that a rope with 12 evenly-spaced knots could form a right-angled triangle. As we know, this 12 is a sum a Pythagorean triple $3 – 4 – 5$. This historian context provided a natural and smooth basis for the investigation activities. Furthermore, the textbook clearly mentioned that the knotted rope context will be used for the next activities, i.e. *'We shall discover the secret of these 'rope-stretchers' in the investigation on pages 210 and 211'*. It is clear that the context in the Singaporean textbook was not just an illustration or additional information. Instead, the context was essential and an integrated part of the content delivery. Moreover, the context of history of mathematics might be interesting for students because they could get a new knowledge on how mathematics has been invented and used since in ancient civilization. The way the textbook invited students' interest is also important because the textbook positioned the knotted ropes as a secret that could be revealed by doing activities in the textbook.
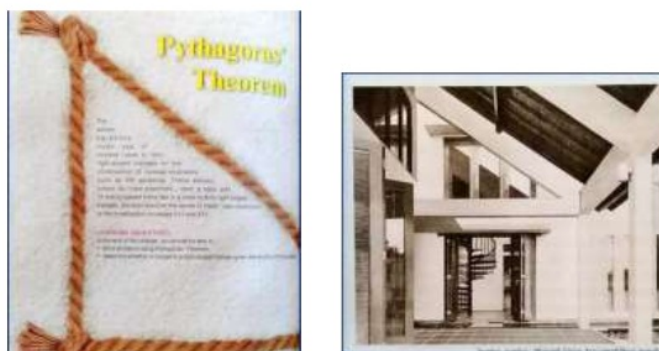

Figure 4. The use of context in the Singaporean textbook (left) and Indonesian textbook (right)
(*source: a scanned page of New Syllabus in Mathematics and Matematika untuk SMP*)

The Indonesian textbook had a different style in selecting the context for introducing the Pythagorean Theorem. This textbook chose a daily life situation or event that was close to students' life, i.e. about

the appearance of right-angled triangles in a building (see the right figure on Figure 5). The context might be interesting for students because it gives students insight about the application of mathematics in daily life and then students grasp the idea that mathematics is not an abstract and isolated concept that is apart from students' real life. However, the use of this context was only at the beginning of the chapter because it was not referred during investigation activities. The investigation activities – i.e. area measurement – was not context-based. Unlike the Singaporean textbook that used an essential context, the Indonesian textbook used context merely as an illustration.

Similar to the Singaporean textbook, the Indonesian textbook also included information about history of mathematics, i.e. in particular about the history of the Pythagorean Theorem (see Figure 5). This information was a supplementary as a reading section and was not as a reference in the other sections of the textbook.
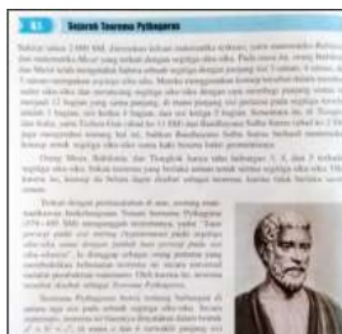

Figure 5. A history of the Pythagorean Theorem in the Indonesian textbook (*source: a scanned page of Matematika untuk SMP*)

With respect to the types of mathematics task, both the Indonesian and Singaporean textbooks included application problems. However, the two textbooks had different approaches in providing the application problems. The application problems in the Indonesian was integrated in the exercise sections and worked-example sections; whereas the Singaporean textbook had a special section containing only application problems. Examples of application problems in the two textbooks can be seen in Figure 6.
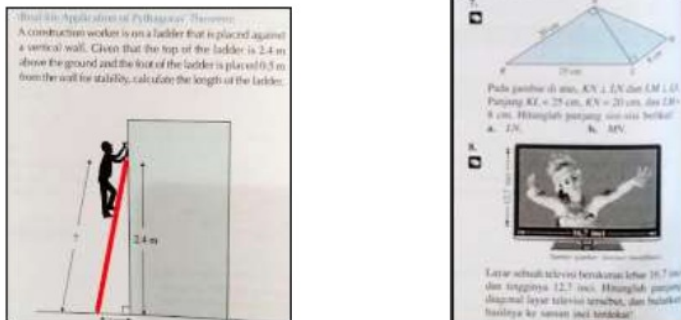

Figure 6. Example of application problem in the Singaporean textbook (left) and in the Indonesian textbook (right)
(*source: a scanned page of New Syllabus in Mathematics and Matematika untuk SMP*)

Lastly, the concern of this present study was the textbooks' attention towards the integration of technology for learning mathematics. With this respect, the Singaporean textbook did not give any

suggestion regarding the use of technology for learning mathematics. A different situation was found in the Indonesian textbook that gave a clear suggestion to use technology for learning mathematics. As shown in Figure 7, the Indonesian textbook gave an alternative for using spreadsheet to check Pythagorean triples. Students were introduced to the use of tables and also conditional formula to efficiently check whether three numbers were Pythagorean triples or not.
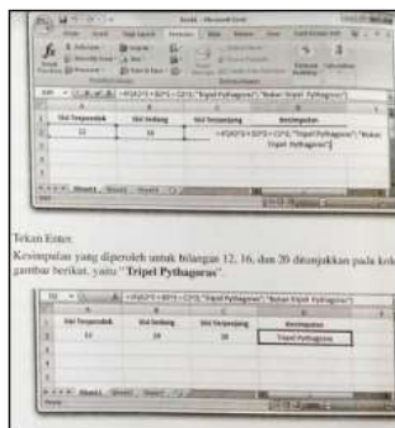

Figure 7. The use of technology for learning mathematics as suggested in the Indonesian textbook
(*source: a scanned page of Matematika untuk SMP*)

4. Conclusions

The present study is aimed to compare the opportunity to learn the Pythagorean Theorem that is provided in Indonesian and Singaporean textbooks. Several similarities and differences were found between the two textbooks. Both the Indonesian and Singaporean textbooks used activity-based explanation. The Pythagorean Theorem was not directly given in the two textbooks, but it was discovered by students through investigation activities. Despite this similar approach for delivering content, the two textbooks used different kind of activities. The Indonesian textbook used area measurement activities, whereas the Singaporean textbook preferred linear measurement activities. Another difference was revealed regarding the use of context. The Singaporean textbook used what De Lange [20] as an essential context because the historian context given at the beginning of the chapter was used a starting point for the investigation activities. In contrast, the context in the Indonesian textbook was not used for the investigation activities. In this way, the context was an illustration and rather a camouflage [20]. Application problems became a concern in both textbooks, but different position was found. The Singaporean textbook had a special section containing only application problems, whereas the Indonesian textbook placed the application problems and bare number problems together in the exercise sections. Last focus of the present study is the attention towards the integration of technology into mathematics learning at which we found suggestion for using computer program is only found in the Indonesian textbook. To conclude, the present study is not aimed to compare the textbooks in terms of deciding which textbook is better than the other one. Instead, the present study explores the similarities and differences between the two textbooks in providing students with opportunity to learn the Pythagorean Theorem.

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